# Unloaded Quality Factor of Transmission Line Resonators With Capacitors

Alexander Zakharov, *Member, IEEE*, and Michael Ilchenko, *Senior Member, IEEE*

***Abstract*—This article contains analytical expressions allowing us to calculate unloaded quality factor $Q_u$ of transmission line resonators with one or several capacitors. These expressions include the $Q$ -factors of the transmission line $Q_l$ and capacitor $Q_c$ and the sensitivity $S_c$ of the main resonance frequency $\omega_0$ of the resonator to small capacitance changing. The sensitivity function $S_c(\omega)$ is a circuit function, which similar to the input and transmission functions. Analysis based on these formulas extended understanding of the character of the resonator unloaded quality $Q_u$ behavior under frequency variation. It is shown that, under certain conditions, connecting additional varactors to the resonator does not increase its unloaded quality factor $Q_u$ . At $Q_c > Q_l$, the value of $Q_u$ of quarter-wave resonator with capacitor decreases with increasing frequency. The established expression for calculate the unloaded quality factor $Q_u$ of resonators with one capacitor allows us to solve the inverse problem and to determine approximately the capacitor quality factor $Q_c$ by measured frequency responses of tunable bandpass filters. Measured results are presented.**

***Index Terms*—Tunable resonator, transmission line, capacitor, resonance frequencies, quality factor, sensitivity of resonance frequencies.**

## I. Introduction

AT PRESENT, there is considerable interest in electrically tunable/reconfigurable filters [1]–[7], despite the fact that tunable filters have been used for a long time [8]–[13]. Such filters are used in diplexers [6], duplexers [7], and as standalone devices [8]–[13]. Most of these filters contain resonators from segments of transmission lines and variable capacitors as tuning elements. Semiconductor varactors [8]–[13], ferroelectric capacitors [14], [15], sets of lumped capacitors commutated by MEMS switches [16] or *pin* diodes [17] are used as variable capacitors. These resonators have a lot of resonant frequencies $\omega_n$, $n = 0, 1, 2, \ldots$. A practical interest is a tuning at a fundamental resonance frequency $\omega_0$. Unloaded quality factor $Q_u$ of such resonators determines the insertion loss ($IL$) of the filters and it variation in the tuning range. This value $Q_u$ depends on the quality factors of transmission line $Q_l$ and capacitor $Q_c$. There are expressions for calculation of $Q_c$ and $Q_l$ [18]. At the same time, there is no formula for determination of $Q_u$ using the known values of $Q_l$ and $Q_c$. This limits the $IL$ controlling of electrically tunable filters. A large number of papers (see, e.g., [10], [13]) are devoted to unloaded quality of resonators $Q_u$ tuned by capacitors; however, a desired formula was not found. Computer simulations have shown that the $Q_u$ values increase as the frequency of the tunable resonator increases [10]. However, this view is somewhat limited.

Resonators consisting transmission line sections and several capacitors are also used [4], [18]–[20]. These resonators are attractive owing to new properties that are not exhibited by resonators with one capacitor. In particular, the effect of intersecting resonance regions is observed in such resonators [4]. One of the main questions not yet answered is the following. Does an additional varactor lead to a significant decrease in the value of $Q_u$? In the literature, analysis results of the unloaded quality $Q_u$ for transmission line resonators with several capacitors are not reported.

In this study, formulas for calculation the unloaded quality $Q_u$ of transmission line resonators with one or several capacitors are founded. These expressions include the $Q$-factors $Q_l$ and $Q_c$ and the sensitivity $S_c$ of the resonance frequency $\omega_0$ of the resonator to small capacitance changing. The sensitivity function $S_c(\omega)$ is a circuit function. It characterizes the local tunability and was introduced in [5]. When forming the $S_c(\omega)$ function, the elements of the sensitivity theory [21]–[24], which has proved itself in many applications, were used. We suggest that analysis based on proposed analytic expressions will allow us to extend the understanding of the character of $Q_u$ variation of the resonator with one or several capacitors in a tuning range. The expressions will also make it possible to determine the quality factor of varactors $Q_c$ from the measured $Q_u$ value, which is of some practical interest.

The article is organized as follows. In section II, the problem of determining the unloaded quality factor $Q_u$ of resonator with one capacitor is solved. In section III, the problem of determining the unloaded quality factor $Q_u$ of resonator with some capacitor is solved. Section IV discusses the problem of determining the quality factor $Q_c$ of ferroelectric capacitors from measured frequency responses of tunable bandpass filters. Our conclusion is given in Section V.

## II. Unloaded Quality Factor $Q_u$ of Resonators With One Capacitor

Let us consider a compound resonator (Fig. 1) formed by a distributed circuit of a lossy transmission line segments and a capacitor with a finite quality factor $Q_c$ connected in parallel.

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The authors are with the Telecommunication Department, National Technical University of Ukraine “Igor Sikorsky Kyiv Polytechnic Institute,” 03056 Kyiv, Ukraine (e-mail: azakharov217@gmail.com).

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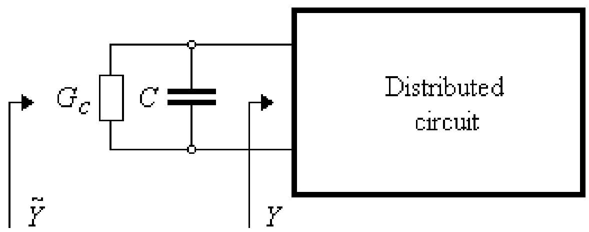



Fig. 1. Resonator containing a distributed lossy circuit and a capacitor with a finite quality factor.

This resonator is equivalent to a one-port network with input admittance $\tilde{Y}$. The one-port network includes capacitance $C$ of the capacitor, its equivalent parallel conductance $G_c$, and distributed circuit with input admittance $Y$. We consider the resonator behavior at fundamental resonance frequency $\omega_0$.

The initial data of the problem are quality factors of the capacitor $Q_c$ and transmission line $Q_l$, which is unloaded quality factor of the resonator from a half-wave or quarter-wave segment of this line. The value $Q_l$ is coupled with the attenuation constant of the transmission line and can be calculated using relations [18]. It is necessary to obtain the formula expressing the unloaded quality $Q_u$ of the compound resonator (Fig. 1) in terms of $Q_c$ and $Q_l$. Let's start with the simplest case, for a lossless transmission line. In this case $Q_l = \infty$.

### A. Lossless Transmission Line

The input admittance of the distributed circuit is purely imaginary: $Y = jB$. The resonance condition for a one-port circuit (Fig. 1) is formulated as zero imaginary part $\tilde{B}$ of its input conductance $\tilde{Y}$

$$\tilde{B} = \omega C + B = 0, \quad B \leq 0. \tag{1}$$

The unloaded quality $Q_u$ of the resonator is determined as follows [23]

$$Q_u = b/G_c \tag{2}$$

where $b$ is the susceptance slop parameter [23]:

$$b = \left.\frac{\omega}{2}\frac{d\tilde{B}}{d\omega}\right|_{\omega=\omega_0}. \tag{3}$$

Taking into account (1) and (3), the expression for $Q_u$ of the resonator in question takes the form

$$Q_u = \frac{\omega C}{2G_c}\left(1 - \frac{dB/d\omega}{B/\omega}\right). \tag{4}$$

Let us transform this expression using resonance frequency sensitivity $S_c$, which was introduced in [5] and has the following sense. Small relative change of the capacitance $\Delta C/C$ results in small relative change of the resonance frequency $\Delta\omega_0/\omega_0$ of the resonator. These changes are coupled as $\Delta\omega_0/\omega_0 = -S_c\,(\Delta C/C)$, where

$$S_c = -\left.\frac{d\omega/\omega}{dC/C}\right|_{\omega=\omega_0} \tag{5}$$

is the sensitivity of resonance frequency $\omega_0$ to the small capacitance changing. For $S_c$ to be positive definite, the minus sign is used in (5). Sensitivity $S_c$ characterizes the resonator tunability.

Using resonance equation (1) we can express the sensitivity $S_c$(5) through the input susceptance $B(\omega)$

$$S_c = \left.\left(1 - \frac{dB}{d\omega}\Big/\frac{B}{\omega}\right)^{-1}\right|_{\omega=\omega_n}, \quad B \leq 0 \tag{6'}$$

If function $B(\omega)$ is given by its zeros $\omega_{0i}$ and poles $\omega_{pi}$, $i = 1, 2, \ldots$(these frequencies are also called critical)

$$B(\omega) = -H\frac{\left(\omega_{01}^2 - \omega^2\right)\left(\omega_{02}^2 - \omega^2\right)\ldots}{\omega\left(\omega_{p1}^2 - \omega^2\right)\left(\omega_{p2}^2 - \omega^2\right)\ldots},$$

then the sensitivity is expressed through these critical frequencies

$$S_c(\omega) = \frac{1}{2}\left[1 - \sum_i \frac{1}{1 - (\omega_{0i}/\omega)^2} + \sum_i \frac{1}{1 - (\omega_{pi}/\omega)^2}\right]^{-1}, \tag{6''}$$

which follows from (6′). Here $H$ is a constant value, called conductivity coefficient, which has no effect on sensitivity $S_c$. A positive-definite function $S_c(\omega)$ is a circuit's function, similar to input susceptance $B(\omega)$. This function is characterized by the following properties:

- the function has only zeros on the frequency axis, no poles;
- it is limited on the entire frequency axis $S_c(\omega) \leq 1/2$;
- its zeros coincide with critical frequencies of the susceptance $B(\omega)$ (6″);
- the function $S_c(\omega)$ is strictly convex between its zeros.

Taking into account expressions for sensitivity (6′) and capacitor quality $Q_c = \omega C/G_c$ [23], we rewrite the expression (4) for the unloaded quality $Q_u$ of the compound resonator in the absence of loss in the distributed circuit. As a result, we obtain

$$Q_u = \frac{\omega C}{2S_c G_c} = \frac{Q_c}{2S_c}. \tag{7}$$

Expression (7) is accurate because it is obtained without any assumptions. It follows from (7) that $Q_u \to \infty$ for $S_c \to 0$, even in the case of small values of $Q_c$.

### B. General Case

If the relationship between $Q$-factors $Q_c$ and $Q_l$ was subject to a widely used pattern

$$\frac{1}{Q_u} = \frac{1}{Q_c} + \frac{1}{Q_l}, \tag{8}$$

then we would have $Q_u = Q_c$ for $Q_l = \infty$. However, this is not the case, since equality (7) is hold. Consequently, it is necessary to use a form of combining $Q_c$ and $Q_l$ different from (8).

As a result, we use the pattern of capacitor's $Q$ factor obtained by parallel or series connection of two other capacitors. If two capacitors with capacitances $C_1$ and $C_2$ and corresponding loss conductance $G_1$ and $G_2$ are connected in parallel, the quality factor $Q$ of the resulting capacitor is determined by the expression [23]

$$\frac{1}{Q}=\frac{G_1+G_2}{\omega\left(C_1+C_2\right)}=\frac{G_1C_1}{\omega C_1\left(C_1+C_2\right)}+\frac{G_2C_2}{\omega C_2\left(C_1+C_2\right)}.$$

Taking into account that

$$\frac{1}{Q_1}=\frac{G_1}{\omega C_1}\quad\text{and}\frac{1}{Q_2}=\frac{G_2}{\omega C_2},$$

we rewrite the previous equality in the form

$$\frac{1}{Q}=\frac{A_1}{Q_1}+\frac{A_2}{Q_2}, \tag{9}$$

where $A_1 = C_1/(C_1+ C_2)$ and $A_2 = C_2/(C_1 + C_2)$ are the weighting coefficients of the corresponding capacitors. Formula (9) expresses the quality factor $Q$ of the resulting capacitor in terms of quality factors $Q_1$ and $Q_2$ of two other capacitors connected in parallel. It can be shown that, in the case of series connection of two capacitors, the quality of the resulting capacitor is also determined by (9). In this case, the weighting coefficients are $A_1 = C_2/(C_1+ C_2)$, $A_2 = C_1/(C_1 + C_2)$. The larger the weighting coefficient, the larger contribution introduced by this element into the resulting quality factor $Q$. The quality factor $Q$ of a composite inductor is also subject to pattern (9).

The weighting (or influence) coefficients $A_1$ and $A_2$ in (9) satisfy the condition

$$A_1+A_2=1. \tag{10}$$

If the quality factors of two capacitors are equal, $Q_1 = Q_2$, the quality factor of the resulting capacitor is the same in the cases of the parallel and series connection. Equalities (9), (10) reflect this regularity. If two inductors are connected, equalities (9) and (10) are also satisfied.

Let us find the solution to the formulated problem in the form (9), which can be written as follows

$$\frac{1}{Q_u}=\frac{A_c}{Q_c}+\frac{A_l}{Q_l}, \tag{11}$$

where $A_c$ is the influence coefficient of capacitor and $A_l$ is the influence coefficient of transmission line segment on the unloaded quality $Q_u$ of the resonator. Formulas (9) and (11) differ from each other by the notation $A_1 = A_c$, $A_2 = A_l$. If one of the terms in (11) is known, the second term can also be found using relationship (10), which solves the formulated problem.

We represent previously set expression (7) in the form

$$\frac{1}{Q_u}=\frac{2S_c}{Q_c},$$

from which we obtain the influence coefficient of capacitor on the quality factor $Q_u$:

$$A_c=2S_c. \tag{12}$$

Since influence coefficient $A_c$ is found (12), from condition (10) we determine the influence coefficient of transmission line segment

$$A_l=1-2S_c. \tag{13}$$

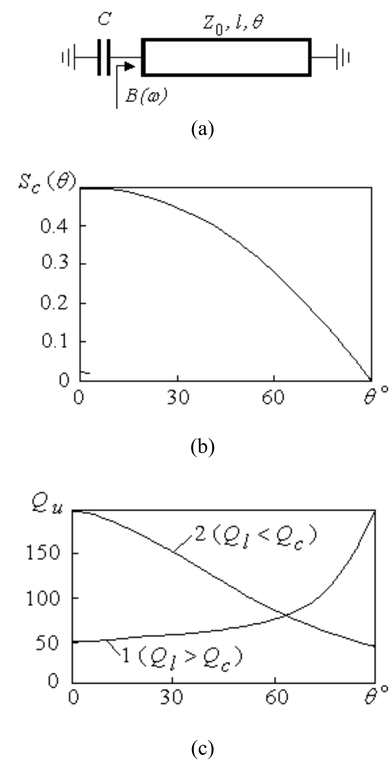



Fig. 2. Combline resonator. (a) Schematic. (b) Sensitivity function $S_c(\theta)$ of the fundamental resonance frequency. (c) The unloaded quality $Q_u$ as a function of its electrical length. Curve *1* corresponds to $Q_l = 200$, $Q_c = 50$, and curve *2* to $Q_l = 50$, $Q_c = 200$.

In [5] it was shown that the sensitivity value is limited: $0 \le S_c \le 1/2$. In turn, this leads to a limited influence coefficients:

$$0\le A_c\le 1;\quad 0\le A_l\le 1.$$

Substituting (12) and (13) into (11), we finally obtain

$$\frac{1}{Q_u}=\frac{2S_c}{Q_c}+\frac{1-2S_c}{Q_l}. \tag{14}$$

If lossless capacitor is used then $Q_c = \infty$ and

$$Q_u=\frac{Q_l}{1-2S_c}.$$

If $S_c = 0$, then $Q_u = Q_l$, but if $S_c = 1/2$, then $Q_u = \infty$.

The values of the sensitivity in (14) are determined by (6′), where $B(\omega)$ is input susceptance of the lossless distributed circuit. This circumstance substantially simplifies the $Q_u$ calculation, since it is not necessary to find the total admittance $Y$ of distributed circuit and separate its imaginary part. Formula (14) is the main result of this study.

### C. Analysis $Q_u$ of Some Resonators

Let us consider two different resonators with a capacitor. The first of them [Fig. 2(a)] is combline resonator, which contains a transmission line segment and capacitor. Here $Z_0$ is the characteristic impedance, $l$ is the segment length, $\theta = \omega l/\upsilon$ is the electric length, $\upsilon$ is the velocity of the electromagnetic wave. The input susceptance of the distributed circuit is determined by the expression

$$B(\omega) = -Z_0^{-1}\cot\theta. \qquad (15')$$

Substituting the susceptance (15′) into (6), we find the expression for the resonator sensitivity function

$$S_c(\theta) = \left(1 + \frac{2\theta}{\sin\ 2\theta}\right)^{-1}. \qquad (15'')$$

The plot of sensitivity (15″) in the region of the main electric length $0 < \theta_0 \leq 90°$ is shown in Fig. 2(b). It can be seen from this figure that the sensitivity increases when electric length $\theta_0$ decreases. This behavior reflects the known fact that the tunability of such resonator at the fundamental resonance frequency increases with decreasing electric length.

Let us analyze unloaded quality $Q_u$ of the resonator for various values of its electric length $\theta_0$ using (14). We consider two variants of values $Q_c$ and $Q_l$: (1) $Q_c = 50$, $Q_l = 200$; (2) $Q_c = 200$, $Q_l = 50$. The plots of $Q_u$ variation for these two variants are shown in Fig. 2(c). Curve *1* corresponds to the first variant, and curve *2* — to the second one. The resonator unloaded quality $Q_u$ increases with increasing $\theta_0$ if $Q_l > Q_c$ (curve *1*). If $Q_l < Q_c$, the value $Q_u$ decreases with increasing $\theta_0$ (curve *2*).

It is reasonable to compare the character of $Q_u$ variation [Fig. 2(c)] and sensitivity $S_c$ [Fig. 2(b)]. In the first case ($Q_l > Q_c$), variations of $Q_u$ and $S_c$ are opposite to each other. In the second case ($Q_l < Q_c$), the unloaded quality $Q_u$ and the sensitivity $S_c$ decrease with increasing electric length. The unloaded quality $Q_u$ is equal to capacitor quality factor $Q_c$ if the electric length tends to zero, and it is equal to line quality factor $Q_l$ for $\theta_0 = 90°$. Therefore, the ratio between $Q_c$ and $Q_l$ determines the increasing or decreasing character of $Q_u$ variation with increasing electric length. For small value of $\theta_0$, the capacitor makes a larger contribution to resonator unloaded quality $Q_u$, and for large $\theta_0$ it has larger influence on the transmission line quality factor $Q_l$. This behavior is due to the influence coefficients in (14)$A_c = 2S$ and $A_l = 1 - 2S$. If $A_c = A_l$, the influences of two elements on $Q_u$ are identical. The equality is achieved if the following condition is satisfied:

$$S_c = 1/4, \qquad (16)$$

which, together with (15″), yields the value of the resonator electric length $\theta_0 = 65.3°$. For electric length $0 < \theta_0 < 65.3°$ ($A_c > A_l$) the capacitor has larger influence on $Q_u$, and, for electric length $65.3° < \theta_0 \leq 90°$ ($A_c < A_l$), the transmission line segment has larger influence on $Q_u$.

The second resonator [Fig. 3(a)] contains the open-ends segment of the transmission line. Its input susceptance and sensitivity function are described by the following functions:

$$B(\omega) = Z_0^{-1}\tan\theta,$$
$$S_c(\theta) = \left(1 - \frac{2\theta}{\sin 2\theta}\right)^{-1}. \qquad (17)$$

The plot of sensitivity (17) in the region of the electric length $90° \leq \theta_0 \leq 180°$ at the fundamental resonance frequency is shown in Fig. 3(b). The function $S_c$ is a bell shaped upward convex function. The maximum of the function $S_{c\max} = 0.1784$ is achieved for $\theta_0 = 129°$. This value is situated in the middle of the optimal band in which tuning is the most efficient.

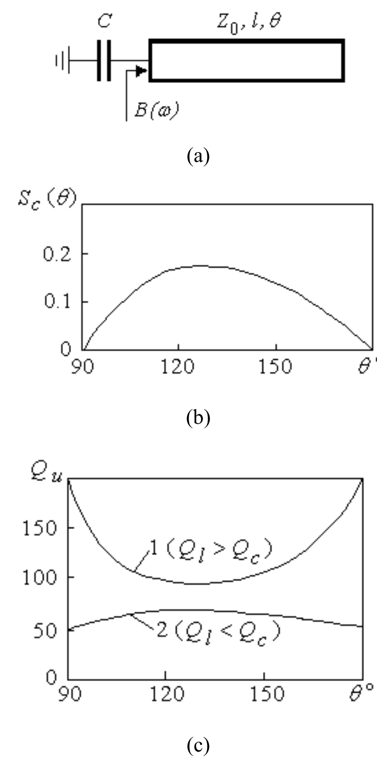



Fig. 3. Transmission line resonator with capacitor and one open end. (a) Schematic. (b) Sensitivity function $S_c(\theta)$ of the fundamental resonance frequency. (c) The unloaded quality $Q_u$ as a function of its electrical length. Curve *1* corresponds to $Q_l = 200$, $Q_c = 50$, and curve *2* to $Q_l = 50$, $Q_c = 200$.

The combline resonator [Fig. 2(a)] has no optimal tuning range. For it, the smaller $\theta_0$, the higher tunability. This is a substantial difference of the two considered resonators. The open-ends resonator exhibits lower tunability than the combline resonator. Its advantage is that it has a larger length at higher frequencies, for example, in the X range. The variation $Q_u$ of open-end resonator is shown in Fig. 3(c) and it has a bell shaped character. As in the case of the combline resonator, we consider two variants of $Q_c$ and $Q_l$ values: (1) $Q_c = 50$, $Q_l = 200$; (2) $Q_c = 200$, $Q_l = 50$. The dependence shown by downward convex curve *1* corresponds to the first case ($Q_l > Q_c$). Upward convex curve *2* is obtained for the second case ($Q_l < Q_c$). In the considered case, as in the previous one, dependence $Q_u(\theta)$ is opposite with respect to sensitivity function $S_c(\theta)$ if $Q_l > Q_c$. If $Q_l < Q_c$, the character of variation of these functions coincides.

Since sensitivity $S_c$ for this resonator is always smaller than 1/4, condition (16) cannot be satisfied. As result, for this resonator the condition $A_l > A_c$ is always satisfied. Therefore, the influence of the line quality factor $Q_l$ on $Q_u$ is always larger than the influence of $Q_c$.

The following specific features of unloaded quality $Q_u$ of considered two resonators were established:

- unloaded quality $Q_u$ of combline resonator for electric length $0 < \theta_0 \leq 90°$ increases with increasing $\theta_0$ if $Q_l > Q_c$;
- unloaded quality $Q_u$ of combline resonator decreases with increasing $\theta_0$ if $Q_l < Q_c$;
- for combline resonator, the influences of $Q_l$ and $Q_c$ on resonator unloaded quality $Q_u$ are identical for electric length $\theta_{0e} = 65.3°$. For electric length $0 < \theta_0 < 65.3°$, the $Q_c$ has larger influence on $Q_u$, and for electric length $65.3° < \theta_0 \leq 90°$ the $Q_l$ has larger influence on $Q_u$;
- the $Q_u$ variation of the open-ends resonator for admissible electric length $90° \leq \theta_0 \leq 180°$ has a bell shaped character. The curve $Q_u(\theta)$ is downward convex for $Q_l > Q_c$ and upward convex for $Q_l < Q_c$;
- for the open-ends resonator, the influence of $Q_l$ on $Q_u$ is always larger than influence of $Q_c$;
- for both resonators, the variation characters of $Q_u(\theta)$ and $S_c(\theta)$ are opposite if $Q_l > Q_c$, and coincide if $Q_l < Q_c$.

Only the first one of the specific features of the resonator quality factor variation is widely known. The other specific features are novel or insufficiently discussed in the literature. They were established with the help of the sensitivity function, introduced in [5].

In capacitance tuning transmission line resonators, the values of $Q_c$ and $Q_l$ are not constant, they change in the tuning range and there are formulas for calculating them. Despite the values of $Q_c$ and $Q_l$ are changed, the sensitivity function $S_c$ (6) and the character of the frequency dependence $Q_u$ are constant (Fig. 2 and Fig. 3).

### D. Verification of $Q_u$ Formula

Formula (7) determining the resonator unloaded quality $Q_u$ in the absence of loss in the transmission line, was rigorously obtained, therefore, its reliability is beyond doubt. The transition from (7) to general expression for unloaded quality $Q_u$ (14) was made under certain assumption. Therefore, (14) requires some additional verification. The values of $Q_l$, $Q_c$ and $S_c$ are determined a priori and introduced into the formula to have calculated $Q_u$ value. The verification can be performed using computer simulation of the *IL* response of the resonator for weak coupling with input and output loads and with the same parameters $Q_l$, $Q_c$ and $S_c$. Simulated *IL* response is used to determine unloaded quality $Q_u$ of the resonator. Such software as Microwave Office (AWR) and HFSS (Ansoft) simulate electromagnetic processes at the electrodynamics level and they are characterized by a high simulation accuracy.

Let us denote *BW* (3 dB) resonator bandwidth by 3 dB level. The value $Q_u$ at resonance frequency $f_0$ is expressed as [24]

$$Q_u = K f_0 / BW(3\text{ dB}), \tag{18}$$

where the coefficient $K$ is determined by the value of insertion loss at resonant frequency $IL_0$

$$K = \frac{10^{-IL_0/20}}{10^{-IL_0/20} - 1}.$$

If $IL_0 = -20$ dB, then we have $K = 10/9$ in (18), and if $IL_0 = -40$ dB, then $K = 100/99$.

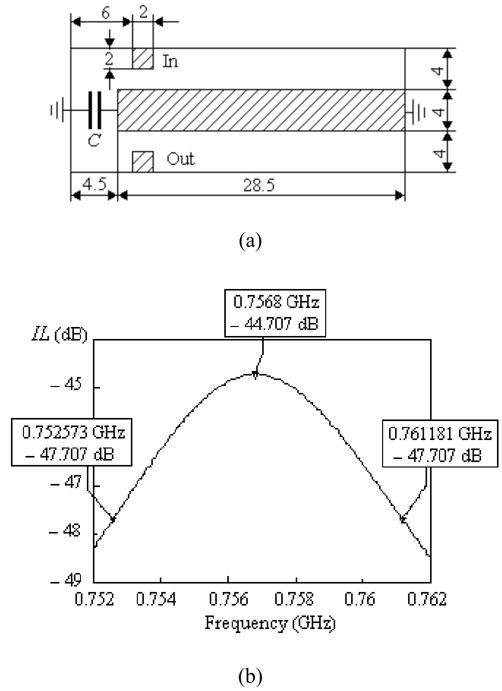



Fig. 4. Definition of unloaded quality factor $Q_u$ of a microstrip combline resonator by EM simulation. (a) The resonator topology in the case of weak coupling with loads. (b) Simulated insertion loss of the resonator with electrical length of 70°.

Fig. 4(a) shows the topology of the combline microstrip resonator in the case of weak coupling with input and output loads. The substrate with the dielectric constant 10.2, the thickness $h = 1.905$ mm, and the height of a metallic screen above the substrate $H = 11.43$ mm are used for this resonator. The width of the microstrip line of the resonator is $w = 4$ mm, which corresponds to $Z_0 = 31.6\ \Omega$. The resonator length is 28.5 mm. Other dimensions are shown in Fig. 4(a) in millimeters. As result, its resonance frequency is 973 MHz at $C = 0$, which corresponds to electric length $\theta_0 = 90°$.

In the simulation, we use the capacitor with $Q_c = 50$ and the microstrip line with $Q_l = 200$. If the dielectric dissipation tan$\delta$ of substrate and air are equal, then the quality factor $Q_l$ of microstrip line is expressed by the equality $Q_l = 1/\tan\delta$. In order to provide the value of $Q_l = 200$, it is necessary to use the dielectric dissipation $\tan\delta = 1/Q_l = 0.005$. The electric length of the resonator $\theta_0$ is controlled by capacitance variation, the larger capacitance $C$, the smaller $\theta_0$. Thus, electric length $\theta_0 = 70°$ corresponds to the resonance frequency 756.8 MHz. In order to obtain these values, the capacitance $C = 2.391$ pF is required. Fig. 4(b) shows the *IL* response of the resonator, which was simulated by Microwave Office software. The results of simulation shown in Fig. 4(b) yield the following values: $IL_0 = -44.707$ dB; *BW* (3 dB) = 8.608 MHz. Substituting these values and $f_0 = 756.8$ MHz into (18) we obtain $Q_u = 88.43$. These data are given in the Table I containing similar values for an electric length 20° and 90° of the resonator. Table I also includes the values of sensitivity $S_c$ and unloaded quality $Q_u$ obtained from (14). The difference between simulated and calculated values of $Q_u$ did

TABLE I
COMPARISON OF RESONATOR QUALITY FACTOR $Q_u$ OBTAINED BY MEANS OF COMPUTER SIMULATED AND CALCULATION

| $\theta_0$, degree | $f_0$, MHz | $C$, pF | $BW$ (3 dB), MHz | $IL_0$, dB | $Q_u$, simulated | $S$ | $Q_u$, calculated | Difference, % |
|---|---|---|---|---|---|---|---|---|
| 90 | 973 | 0 | 4.991 | – 31.35 | 200.4 | 0 | 200 | 0.2 |
| 70 | 756.8 | 2.391 | 8.608 | – 44.707 | 88.43 | 0.21 | 88.5 | 0.08 |
| 20 | 216.2 | 61.85 | 4.211 | – 58.144 | 51.4 | 0.479 | 51.63 | 0.44 |

not exceed 0.5%. The comparison confirms the reliability of the general expression (14) for unloaded quality $Q_u$.

## III. Unloaded Quality $Q_u$ of Resonators With Some Capacitors

Consider resonators containing $m$ capacitors with capacitance $C_i$, $i = 1, 2, \ldots m$ that are characterized by quality factors $Q_{ci}$. Denote the input susceptance $B_i$ at the connection point of $C_i$. Using (6), it is possible to calculate the sensitivity $S_{ci}$ of the resonant frequency to the capacitance change of this capacitor.

### *A. Formula for $Q_u$ Calculation*

Since the considered resonators are linear circuits, (14) for resonators with one capacitor can be extended to resonators with several capacitors. It takes the form

$$\frac{1}{Q_u} = 2\sum_{i=1}^{m} \frac{S_{ci}}{Q_{ci}} + \frac{1 - 2\sum_{i=1}^{m} S_{ci}}{Q_l}. \tag{19}$$

In (19) the quantities

$$A_{ci} = 2S_{ci}, \quad A_l = 1 - \sum_{i=1}^{m} 2S_{ci} \tag{20}$$

are, respectively, the influence coefficient of the capacitor with number $i$ on quality factor $Q_u$ of the resonator, and the influence coefficient of the transmission line on $Q_u$. The greater coefficients (20), the more substantial the influence of the corresponding elements on $Q_u$.

Influence coefficients (20) have the following restrictions:

$$0 \le \sum_{i=1}^{m} A_{ci} \le 1; \quad 0 \le A_l \le 1; \quad A_l + \sum_{i=1}^{m} A_{ci} = 1$$

which follow from the more general condition [5]

$$\sum_{i=1}^{m} S_{ci} \le 1/2. \tag{21}$$

The equation from condition (21) is fulfilled for lumped $LC$ circuits in which all capacitances are variable. This equality is known [21], [22] as invariant of sensitivity.

For a resonator with several capacitors, the condition for the equal influence coefficients on the unloaded quality $Q_u$ of the resonator is represented as

$$\sum_{i=1}^{m} S_{ci} = 1/4. \tag{22}$$

This condition follows from equality of the influence coefficients (20): $\sum A_{ci} = A_l$. Condition (16) is a special case of condition (22) at $m = 1$.

Formula (19) enables us to state the following.

*Statement.* If two resonators contain a section of one transmission line and different numbers $m_1$ and $m_2$ of capacitors with equal $Q$ factor, these resonators have equal unloaded quality $Q_{u1} = Q_{u2}$ at the same frequencies when the sums of sensitivities are equal,

$$\sum_{i=1}^{m_1} S_{ci} = \sum_{j=1}^{m_2} S_{cj}. \tag{23}$$

*Proof:* Since the $Q$ factors of capacitors are equal, $Q_{ci} = Q_{cj} = Q_c$, the substitution of $Q_c$ into (19) yields the expressions for the unloaded quality of these resonators $Q_{u1}$ and $Q_{u2}$:

$$\frac{1}{Q_{u1}} = 2\sum_{i=1}^{m_1} \frac{S_{ci}}{Q_c} + \frac{1 - 2\sum_{i=1}^{m_1} S_{ci}}{Q_l},$$

$$\frac{1}{Q_{u2}} = 2\sum_{j=1}^{m_2} \frac{S_{cj}}{Q_c} + \frac{1 - 2\sum_{j=1}^{m_2} S_{cj}}{Q_l}.$$

Since condition (23) is fulfilled, the right-hand sides of these expressions are identical, which yields the equality $Q_{u1} = Q_{u2}$. The statement is proved.

Expression (19) does not contain capacitances $C_i$ but takes into account only the degree of their influence on the resonance frequency expressed by sensitivity $S_{ci}$. The statement proved above indicates that the unloaded quality of two resonators can coincide for different numbers of capacitors and with different values of their capacitances.

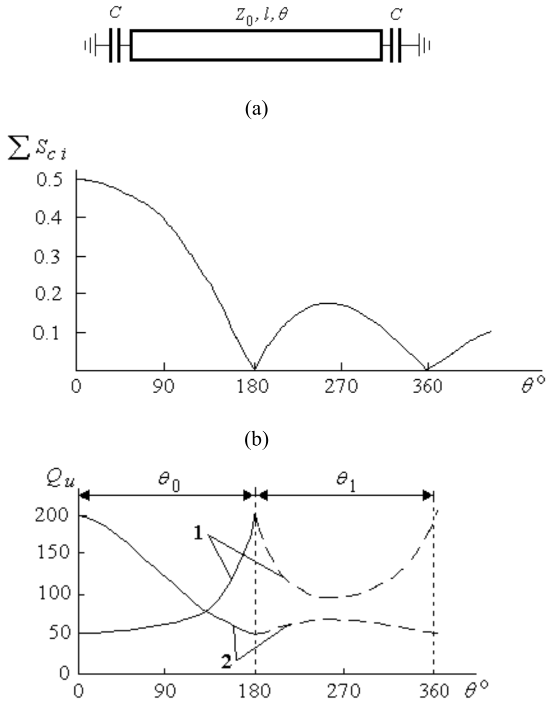
We use (19) to analyze certain resonators.

### *B. Transmission Line Resonator With Two Capacitors*

Let us analyze a resonator consisting of a transmission line section and two identical capacitors [Fig. 5(a)] whose capacitances $C_1$ and $C_2$ are equal and vary synchronously. The $Q$ factors of the capacitors are also equal, $Q_{c1} = Q_{c2} = Q_c$. The transmission line section has characteristic impedance $Z_0$. Its geometric length is $l$, and the electric length is $\theta$. When the capacitances of the capacitors are zero, we have a half-wave resonator. The resonance frequencies of the considered resonator are determined from two equations [4]:

$$\omega_n C = Z_0^{-1} \cot(\theta_n/2), \qquad \omega_n - \text{even}; \tag{24}$$

$$\omega_n C = -Z_0^{-1} \tan(\theta_n/2), \qquad \omega_n - \text{odd}. \tag{25}$$




(c)

Fig. 5. Transmission line resonator with two capacitors. (a) Schematic. (b) Sensitivity of the resonance frequencies $\omega_0$ and $\omega_1$ to changing of two capacitances. (c) The unloaded quality $Q_u$ of the resonator as a function of its electrical length. The solid (dashed) lines correspond to the frequency $\omega_0$ ($\omega_1$). Curves *1* for the case $Q_l = 200$ and $Q_c = 50$. Curves *2* for the case $Q_l = 50$ and $Q_c = 200$.

It follows from (24) and (25) that the resonance electric lengths $\theta_n$ of this resonator satisfy the condition $(n-1)\pi \leq \theta_n \leq n\pi$. The width of all resonant regions is the same and equal to $\pi$.

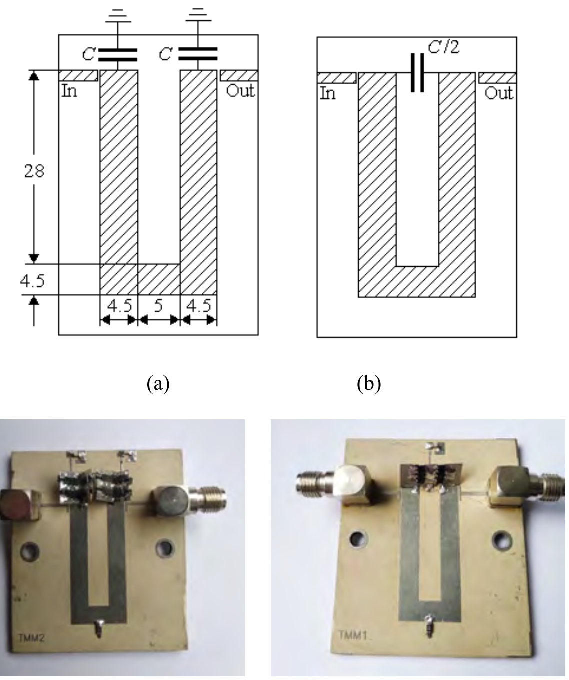
The substitution of (24) and (25) into (6) yields the total sensitivity of resonance frequency $\omega_n$ to synchronous variation of two capacitances of the resonator:

$$S_{c1} + S_{c2} = 2S_c = \left(1 + \frac{\theta}{\sin\theta}\right)^{-1}, \qquad \omega_n - \text{even}; \quad (26)$$

$$S_{c1} + S_{c2} = 2S_c = \left(1 - \frac{\theta}{\sin\theta}\right)^{-1}, \qquad \omega_n - \text{odd}. \quad (27)$$

The plots of sensitivities (26) and (27) are depicted in Fig. 5(b) for the initial resonance frequencies. As the electric length grows, the sum of the sensitivities at fundamental resonance frequency $\omega_0$ monotonically decreases from the maximum value 1/2 to zero at $\theta = 180°$. The curves of the total sensitivity of resonance frequencies $\omega_1$ and $\omega_2$ are bell-shaped.

To analyze unloaded quality $Q_u$ of the considered resonator, we use two variants of the values of the $Q$ factors: 1. $Q_l = 200$ and $Q_c = 50$; 2. $Q_l = 50$ and $Q_c = 200$. Fig. 5(c) shows the plots of the variation of $Q_u$ at resonance frequencies $\omega_0$ and $\omega_1$. At frequency $\omega_0$, equality (22) is fulfilled for the electric length $\theta_0 = 130.6°$, at which the influence of the capacitors and the transmission line on unloaded quality $Q_u$ are identical.




(c) (d)

Fig. 6. Fabricated tunable microstrip hairpin resonators. (a) Topology of the resonator with two capacitors. (b) Topology of the resonator with one capacitor. (c) Photograph of the resonator with two capacitors. (d) Photograph of the resonator with one capacitor.

In the interval of electric lengths $0 < \theta_0 < 130.6°$, the capacitors more substantially affect $Q_u$, and the transmission line section more substantially affects $Q_u$ in the interval of electric lengths $130.6° < \theta_0 \leq 180°$. At frequency $\omega_1$ the total sensitivity is below 1/4, and, therefore, the transmission line section always more substantially affects $Q_u$.

At all cases in Fig. 5 the variations of unloaded quality $Q_u(\theta)$ and total sensitivity ($S_{c1}+ S_{c2}$) have opposite characters when $Q_l < Q_{c1}$, $Q_{c2}$. If $Q_l < Q_{c1}$, $Q_{c2}$, the characters of variations of these functions coincide.

### C. Experimental Verification of Statement

The concept formulated in the statement is important for practice. Consider it in more detail using experimental verification that enables us to directly determine the unloaded quality factor $Q_u$ of resonators. Fig. 6 displays the topologies of two hairpin microstrip resonators with capacitors. The characteristic impedance of microstrip lines is $Z_0$, and their electric length is $\theta$.

Fig. 6 shows the topologies and photographs of hairpin microstrip resonators with two and one capacitors, which have a weak coupling with loads. The dimensions are in millimeters. In the microstrip resonators a substrate made of the RT/Duroid 6010LM laminate (produced by Rogers company) with $\varepsilon_r = 10.2$, $h = 1.905$ mm, and $\tan\delta = 0.002$ was used. The width of microstrip line of hairpin resonator $w = 4.5$ mm ($Z_0 = 29.3\ \Omega$). To provide applying the control voltage to the varactors of the hairpin resonators, a 10 k$\Omega$ chip resistor is connected to the middle of these resonators [Figs. 6(c) and 6(d)].

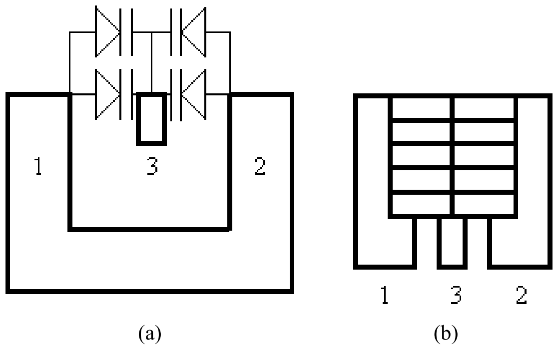



Fig. 7. Unit with the set of varactors that used in the resonators. (a) Schematic with four varactors. (b) One side of the unit with varactors.

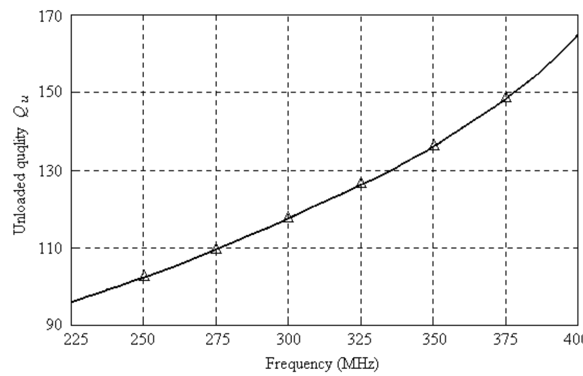



Fig. 8. Measured unloaded quality factor $Q_u$ of tunable hairpin resonators in Figs. 6(c) and 6(d).

A resonator with two capacitors $C$ was considered above (Fig. 5), and now it has adopted a hairpin shape [Fig. 6(a)]. Its main resonance frequency $\omega_0$ is determined by the resonance equation (24) and the total sensitivity of $\omega_0$ to changes in two capacitances is expressed by (26). The resonator in Fig. 6(b) contains only one capacitor, the value of which is $C/2$. It has the same resonance frequency $\omega_0$ as the previous resonator because it is described by the same resonance equation (24) with a multiplier (1/2) in its right part [3]. The sensitivity of $\omega_0$ of this resonator to changes in one capacitance is the same as the sensitivity of the previous resonator to changes in two capacitances (26). It follows from the coincidence of the resonance equations.

When all capacitances of the resonators are zero, their resonance frequency is $f_0 = 850$ MHz. This value corresponds to the electric length of the resonators $\theta_0 = 180°$. We will tune these two resonators from $f_{0\ \min} = 225$ MHz to $f_{0\ \max} = 400$ MHz, which corresponds to a change in their electric length from $\theta_{0\ \min} = 47.64°$ to $\theta_{0\max} = 84.7°$. Substituting these values of $\theta_0$ in (24) we determine the boundary values of tuning capacitance: $C_{\min} = 14.9$ pF; $C_{\max} = 54.68$ pF.

In the loop hairpin resonator [Fig. 6(b)] the value of a single capacitance changes within the range from $C/2_{\min} = 7.45$ pF to $C/2_{\max} = 27.34$ pF. To implement the required values of capacitance $C/2 = 7.45$–$27.34$ pF we use several varactors BB135 (NXP company) with minimal capacitance $C_{\min} \approx 2$ pF, which are connected in parallel and in series and form a unit. In Fig. 7(a), a loop hairpin resonator with the unit consisting of two pairs of oppositely connected varactors is presented. Its capacitance is the same as of a single varactor. Controlling voltage is applied to the interior of the unit through the additional metallized plate *3*, disposed between the arms *1* and *2* of the resonator.

In Fig. 7(b) the structure of the unit for assembling a few metal strips are denoted by numbers *1*, *2*, and *3*. The varactors are soldered to both sides of these strips. Such a unit has small dimensions. For example, a unit with dimensions 7 mm × 6 mm can be used for placing of 10 pairs of varactors in SOT23 packages. The varactors are mounted on both sides of the unit. After that, this unit is soldered in a vertical position to the left *1* and the right *2* parts of the loop hairpin resonator, as well as to its additional metal plate *3* [Fig. 7(a)].

The unit must contain 15 varactors BB135 in order to change the $C/2$ capacitance from $C/2_{\min} = 7.45$ pF to $C/2_{\max} = 27.34$ pF, and bias voltage should vary from 6 V to 27.2 V. The photograph of the loop hairpin resonator with one unit is shown in Fig. 6(d). The hairpin resonator with two capacitors [Fig. 6(a)] contains four units and his photograph is shown in Fig. 6(c).

Fig. 8 shows the measured unloaded quality $Q_u$ of the two tunable hairpin resonators in the frequency range 225-400 MHz. The $Q_u$ values of both resonators are the same, and they change from $Q_u = 98$ (225 MHz) to $Q_u = 167$ (400 MHz). The expression (18) was used to obtain the dependence $Q_u = Q_u(f_0)$. Resonator in Fig. 6(d) contains 15 varactors, and resonator in Fig. 6(c) contains 60 varactors. The use of a large number of varactors averages the spread of their parameters and makes measurements more accurate.

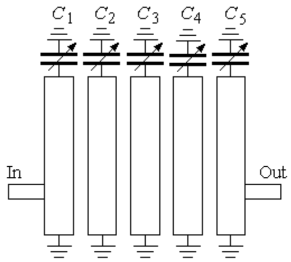
The transition from the resonator shown in Fig. 6(d) to the resonator shown in Fig. 6(c) has an important feature. Resonator in Fig. 6(d) contains one capacitor with capacitance $C/2$, and the resonator in Fig. 6(c) has two capacitors, each of them has a capacitance $C$. In this case the unloaded quality $Q_u$ of both resonators is the same.

The considered measured data confirm the correctness of the above statement and formula (19) for unloaded quality $Q_u$ of a resonator containing several capacitors.

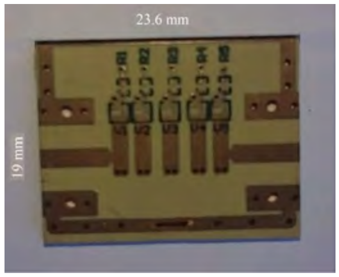
It was established in [5] that if $S_c$ values is quite large ($S_c \geq 0.4$), then the use of additional varactors will not lead to a significant increase in the tunability of resonator. Now we can say that this will not lead to a significant reduction in unloaded quality $Q_u$ of the resonator either.

## IV. Definition of Quality Factor $Q_c$ of Ferroelectric Capacitors

An important problem is the determining of quality factor $Q_c$ of ferroelectric capacitors from a measured frequency response of tunable bandpass filters (BPFs). This problem can be solved by using the obtained equation (14). The quality factors of ferroelectric capacitors $Q_c$ based on barium strontium titanate (BST) films have been determined below. The capacitors were used in four BPFs tuned over the frequency range from 1.8 to 4.3 GHz.




(a)




(b)

Fig. 9. Tunable microstrip filter with ferroelectric capacitors. (a) Schematic. (b) Photograph without shield.

### A. Characteristics of Tunable BPFs

Let us consider four fifth-order combline BPFs [Fig. (9a)] that are tuned by BST capacitors. The filters are made on Rogers TMM-3 substrate with the dielectric constant $\varepsilon_r = 3.27$ and $\tan\delta = 0.002$. The substrate thickness $h = 0.762$ mm, the height of the metal shield above the substrate $H = 4$ mm. One end of combline resonator is short-ended, and the other is loaded with a BST capacitor with capacitance $C$. The resonance frequencies of combline resonators are determined by (15′).

Fig. 9(b) shows the photograph of one of the filters without shield. All filters have the same dimension 23.6 mm × 19 mm × 5 mm. The width of the central conductor of the microstrip resonators is $w = 1.4$ mm, and their characteristic impedance is $Z_0 = 57.4\ \Omega$. The four filters differ from each other in the length of the resonators $l_1 = 13$ mm, $l_2 = 10$ mm, $l_3 = 8.4$ mm, and $l_4 = 7$ mm. Short circuit at the ends of the resonators is provided by metalized holes with the diameter 0.4 mm. The voltage applied to the capacitors was varied from 5 V to 100 V. Fig. 10 shows the insertion loss of the filters. Curves *1*,*2*,*3* correspond to a voltage of 5 V, 50 V, and 100 V, respectively.

A change in the bias voltages from 5 to 100 V resulted in tuning of the first filter [Fig. 10(a)] in a frequency range from $f_{0\ \min} = 1778$ MHz to $f_{0\ \max} = 2320$ MHz, which corresponds to the fractional tuning range FTR $= 2(f_{0\ \max} - f_{0\ \min})/(f_{0\ \max} + f_{0\ \min}) = 0.26$. The capacitance of the BST capacitors was varied from 0.76 pF to 1.6 pF, which corresponds to the capacitance ratio $C_{\max}/C_{\min} = 2.1$. At a voltage of 5 V, the center frequency of the first filter $f_{0\ \min} = 1784$ MHz, the 3-dB bandwidth $BW(3\text{ dB}) = 218$ MHz, which corresponds to the fractional bandwidth $FBW = BW/f_0 = 0.122$, and the minimum passband insertion loss $IL_0 = 4.26$ dB. At a voltage of 100 V, the filter's center frequency $f_{0\ \max} = 2320$ MHz, the bandwidth $BW(3\text{ dB}) = 273$ MHz, which corresponds to $FBW = 0.118$, and $IL_0 = 3.24$ dB. These values, which characterize the first filter, are listed in Table II. The similar values for the other three filters, and the values of the return loss $RL$ and the resonator electrical lengths $\theta$ are included in Table II also. This table uses the calculated values of the variable capacitance obtained in the design process. Note that these values are not used to determine the unloaded quality factor $Q_u$ (14). They are presented in this table for a more complete description of the filters.

TABLE II
CHARACTERISTICS OF MICROSTRIP FIFTH ORDER BPFS WITH FERROELECTRIC CAPACITORS

| Characteristics | Filter number | | | |
|---|---|---|---|---|
| | 1 | 2 | 3 | 4 |
| $f_{0\min}$–$f_{0\max}$ (MHz) | 1784-2320 | 2306-2851 | 2710-3490 | 3477-4333 |
| FTR (%) | 26 | 21 | 25 | 22 |
| $C_{\min}$–$C_{\max}$ (pF) | 0.76-1.6 | 0.7-1.26 | 0.54-1.08 | 0.4-0.76 |
| $C_{\max}$ / $C_{\min}$ | 2.1 | 1.8 | 2.0 | 1.9 |
| $FBW_{\max}$ (%) | 11.8-12.2 | 14.5-16.1 | 13.8-14.2 | 12.7-13.3 |
| $IL_{0\min}$–$IL_{0\max}$ (dB) | 3.24-4.26 | 3.03-4.26 | 3.14-5.3 | 3.15-5.63 |
| $RL_{\min}$–$RL_{\max}$ (dB) | (12-21) | (11-13) | (13-20) | (12-17) |
| $\theta^o_{\min}$–$\theta^o_{\max}$ | 44.1º -57.4º | 44º-54.4º | 43.4º-55.9º | 46.5 º -58º |

We will solve the problem of determining the quality factor of the ferroelectric capacitors $Q_c$ from the measured frequency responses of the filters in two stages. At the first stage we will find the unloaded quality $Q_u$ of the filter's resonators, which include transmission line segments and ferroelectric capacitors. At the second stage, based on the obtained values of $Q_u$, we will determine the quality factor $Q_c$ of the ferroelectric capacitors.

### B. Unloaded Quality $Q_u$ of Tunable Resonators

In designing BPFs, the Chebyshev, or equal-ripple, attenuation characteristics of lossless filters are most often used [25]. Suppose that $N$-order BPF should be design with ripple $L_{Ar}$, midband frequency $f_0$ and bandwidth BW. We note that the value of BW is determined by the $L_{Ar}$ level. The normalized $g$-parameters of lowpass prototype filters are used for the filter design. The insertion loss of lossless BPF with odd number of resonators is zero at the midband frequency $IL_0 = 0$. If all resonators of BPF have the same unloaded quality $Q_u$, the filter midband insertion loss is determined by the known formula [25]

$$\Delta L_{A0} = 4.343\, f_0 \frac{\sum_{k=1}^{N} g_k}{Q_u BW}\ (\text{dB}), \tag{28}$$

where $\Delta L_{A0}$ is increase of insertion loss at $f_0$.

We have to solve the inverse problem of determining the resonators' unloaded quality $Q_u$ from the measured filter insertion loss (Fig. 10). To this end, we must determine some values in (28). The main difficulty in solving the inverse problem is due to the low unloaded quality $Q_u$, a circumstance that results in significant variations in the filter insertion loss in the pass band region. It seems impossible through direct measurements the lossless value of $BW$ and ripple level $L_{Ar}$, which corresponds to $g_k$ sum in (28).

To solve this problem, it is appropriate to use the filter characteristics that are least affected by the dissipative loss. Among these characteristics are return loss $RL$ and width of

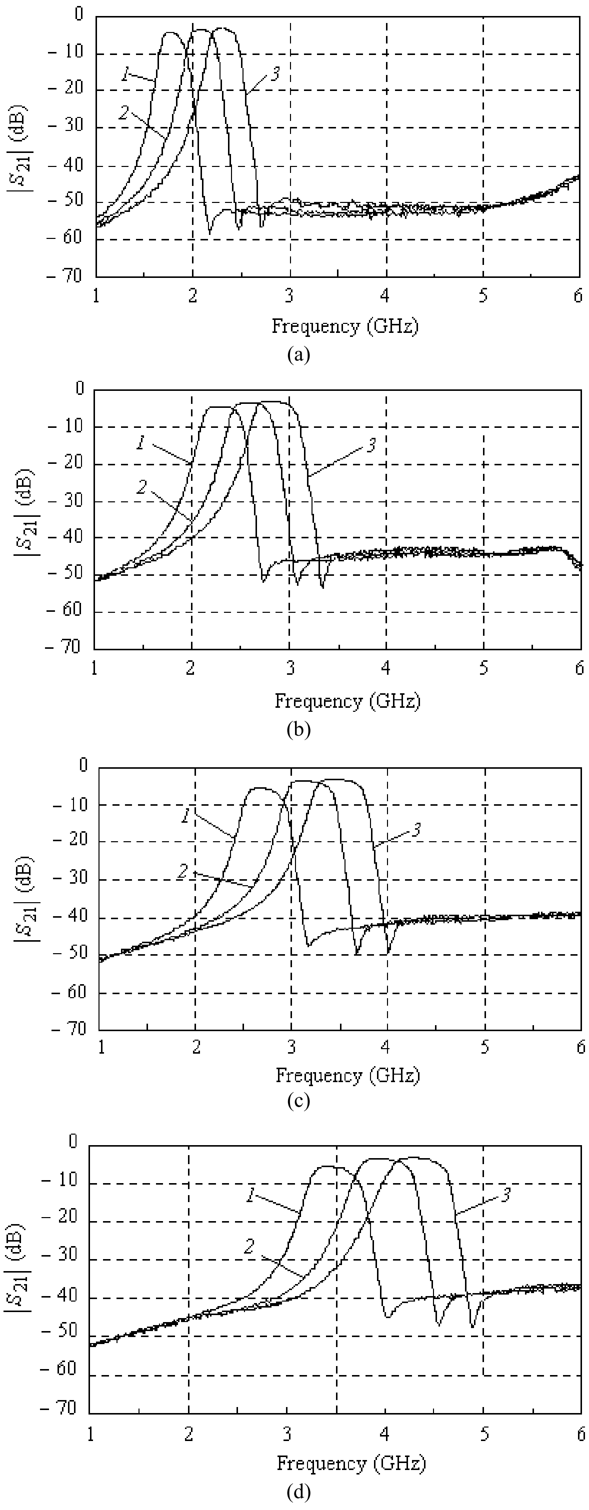



Fig. 10. Insertion loss of microstrip filters with ferroelectric capacitors, operating in different frequency ranges. (a) 1.78-2.32 GHz. (b) 2.30-2.82 GHz. (c) 2.71-3.49 GHz. (d) 3.47-4.33.

“barrier band” $\Delta f_b$. The “barrier band” is passband width measured by high attenuation level. The values $RL$ and $\Delta f_b$ we determine by the measured frequency responses. The transition from $\Delta f_b$ to lossless $BW$ is performed with the use of the filter’s insertion loss plots [25]. Knowing $RL$ we can determine

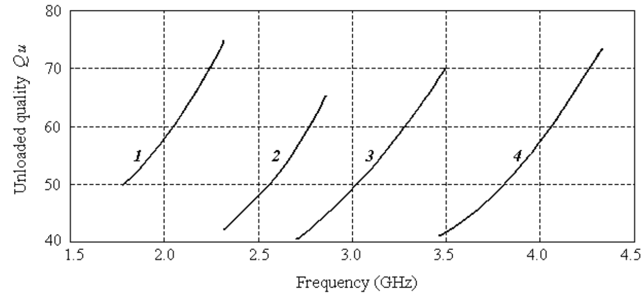



Fig. 11. Unloaded quality $Q_u$ of the combline resonator with ferroelectric capacitor over the tuning range of each BPF.

the ripple level [18]

$$L_{Ar} = -10 \lg (1 - 10^{RL/10}) \text{dB}. \tag{29}$$

The above solution allows us to roughly define the unloaded quality $Q_u$ of the resonators. The basis for the design of these filters is formed by the Chebyshev characteristics. Let us assume that the averaged return loss of the filters $RL = 13.5$ dB (Table II). Using (29), we find the ripple level corresponding to this value of return loss $L_{Ar} = 0.2$ dB. Next we find the sum of $g$-parameters for a fifth-order Chebyshev filter with $L_{Ar}\,0.2$ dB [25]: $\sum g_k = 7.52$.

For the first BPF at the lower center frequency $f_{0\min} =$ 1784 MHz [Fig. 10(a), curve *1*], the measured 20 dB “barrier band” width is $\Delta f_b = 400$ MHz. From the insertion loss curve [25], for $L_{Ar} = 0.2$ dB and $N = 5$, we find the ratio $\Delta f_b/BW = 1.44$. The $BW$ of a lossless filter is determined as $BW = 400/1.44 \approx 278$ MHz. The measured insertion loss of the BPF at $f_0$ is $IL_0 = \Delta L_{A0} = 4.26$ dB. Substituting the found values $\sum g_k = 7.52$, $BW = 278$ MHz, $\Delta L_{A0} =$ 4.26 dB, $f_0 = 1784$ MHz into (28), we obtain $Q_u \approx 49.2$.

For the same filter, at the upper center frequency $f_{0\max} =$ 2320 MHz [Fig. 10(a), curve *3*], we find the 20-dB “barrier band” width: $\Delta f_b = 450$ MHz. Next we turn to the lossless $BW = 312$ MHz. Taking into account the measured insertion loss $\Delta L_{A0} = 3.24$ dB, we find $Q_u \approx 75$ from (28). Curve *1* in Fig. 11 shows the resonator $Q_u$ variation of the first BPF over its tuning range. The values of $Q_u$ vary from 49.2 to 75. Other three curves in Fig. 11 correspond to another three BPFs from Table II.

Determining $Q_u$ we arbitrarily assumed that the averaged level of the filter return loss $RL = 13.5$ dB. Let us analyze the possible results of a change in this level. If $RL = 16$ dB, then $L_{Ar} = 0.1$ dB, $\sum g_k = 7.01$, and $\Delta f_b$/BW $= 1.52$. If we take $RL = 9.5$ dB, then $L_{\text{Ar}} = 0.5$ dB, $\sum g_k = 8.42$, and $\Delta f_b/BW = 1.35$. In the above considered cases, the unloaded quality $Q_u$ calculated with (28) will differ from the prior values $Q_u$ corresponding to $RL = 13.5$ dB. In the first case, they will be underestimated by 2%, and in the second case, they will be overestimated by 5%. In actual practice, this accuracy is acceptable.

### *C. Quality Factor $Q_c$ of Ferroelectric Capacitors*

Let us consider the second stage of solving the problem of obtaining quality factor of the ferroelectric capacitors $Q_c$

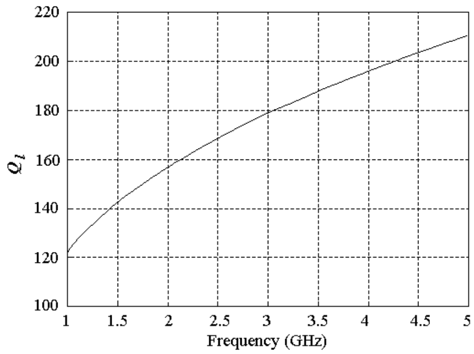



Fig. 12. Quality factor of a microstrip transmission line with width $w = 1.4$ mm made on TMM-3 substrate with thickness $h = 0.761$ mm.

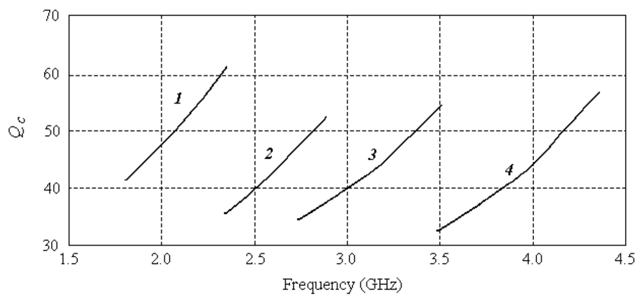



Fig. 13. Variation in quality factor $Q_c$ of the ferroelectric capacitor over the tuning range of each BPF.

from the found values of resonator's unloaded quality $Q_u$ and calculated quality factor $Q_l$ of the resonators' transmission line. For the microstrip transmission line used in the resonators under consideration, these calculated formulas [18] give the values of $Q_l$, which plotted in Fig. 12.

Now, we have the values of $Q_u$ and $Q_l$ (Fig. 12). On the basis of these values, it is necessary to determine the quality factor $Q_c$ of the ferroelectric capacitors. To solve this problem, we use (14), which determines the unloaded quality $Q_u$ of a composite resonator containing a transmission line segment and a capacitor.

For the first BPF at the lower center frequency $f_{0\min} = 1784$ MHz, the known values are $Q_u = 49.2$, $Q_l = 151$, and $\theta = 44.14°$. At the specified electric length, using (15″), we find the sensitivity $S_c = 0.393$. Substitution of these values $Q_u$, $Q_l$, and $S_c$ into (14) gives the quality factor of the ferroelectric capacitor $Q_c \approx 41.6$.

Computing $Q_c$ we use the calculated quality factor of the microstrip transmission line $Q_l = 151$. The formulas for calculating $Q_l$ [18] do not take into account the roughness of the dielectric substrate and the loss in the short-ended circuit of the resonator. We assume that these effects will reduce the estimated $Q_l$ by about 30%, i.e., $Q_l = 106$, and repeat the previous calculations. As a result, we obtain $Q_c \approx 42.9$. Thus, as $Q_l$ is decreased by 30%, the calculated quality factor of the ferroelectric capacitor increases from 41.6 to 42.9, i.e., by 3%. Note that the roughness of the substrate does not affect the quality factor of capacitor $Q_c$ soldered onto the substrate.

Determining $Q_c$ we use the values of $Q_l$ reduced by 30%. Determining $Q_c$ at the upper center frequency $f_{0\max} = 2320$ MHz we use the following values $Q_u = 75$, $Q_l = 123$, $\theta = 57.4°$, and $S = 0.312$. Substitution of these values into (14) gives $Q_c = 55.7$.

The plots of the quality factor of the ferroelectric capacitors $Q_c$ over the same tuning range for each BPF are shown in Fig. 13. Over the tuning range from 1.8 to 4.3 GHz the quality factor of the ferroelectric capacitors under consideration take the values $Q_c = 33.5$–$60.7$.

## V. Conclusion

Analytical expressions allowing to determine unloaded quality $Q_u$ of transmission line resonators with one (14) and several (25) capacitors are established. The basis of the obtained expressions is the sensitivity $S_c$ of resonant frequency of resonators to changes in capacitances of these capacitors. In addition to the sensitivity $S_c$ these expressions include the quality factors of varactors $Q_c$ and transmission line $Q_l$. The analysis performed on the basis of these formulas expands the understanding of the nature of $Q_u$ change of resonators in the tuning range. The expression (14) allows us to solve the inverse problem and to determine approximately capacitor quality factor $Q_c$ by measured frequency responses of tunable bandpass filters.

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**Alexander Zakharov** (Member, IEEE) received the M.S. degree in physics from Kiev State University, Ukraine, in 1971, the Ph.D. degree in radio engineering from the National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute," in 1986, and the D.Sc. degree in solid state electronics from the Cybernetics Institute of Academy of Science, Ukraine, in 1994. From 1971 to 1988, he was a Lecturer with the Kiev Aviation College. He joined as a Researcher with the Telecommunication Department, National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute," in 1988, where he is currently the Head Scientist. His current research interests include circuits' theory, RF/microwave filters and duplexers for telecommunication systems, and non-uniform transmission line resonators.

**Michael Ilchenko** (Senior Member, IEEE) received the M.S. and Ph.D. degrees in radio engineering from the National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute" in 1964 and 1967, respectively, and the D.Sc. degree in solid state electronics from the Cybernetics Institute, National Academy of Sciences at Ukraine (NASU), Ukraine, in 1980. He was as a Senior Researcher and an Assistant Professor with the National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute," where he has been a Full Professor since 1982 and a Pro-Rector for Research since 1988. He has authored or coauthored several books, the one of book, namely, *Dielectric Resonators* (1989), is available with the U.S. Library of Congress. His current interest includes microwave technologies in telecommunications systems. Since 1997, he has been an Active Member (Academician) with NASU. He was a recipient of three Ukrainian State Prizes and three awards for Science and Technology Excellence.